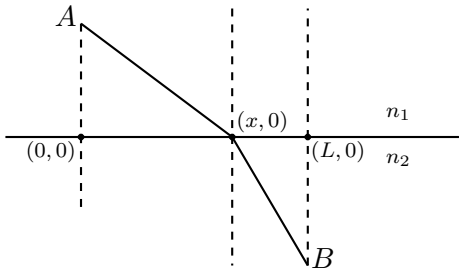
PHY 71100: ANALYTICAL DYNAMICS

Problem Set 1

Due September 16, 2024

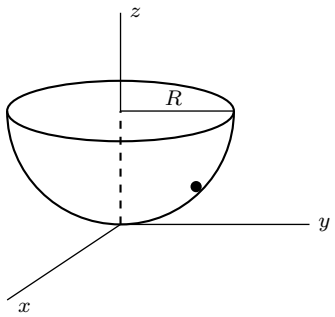
Problem 1

Consider two optically different materials with indices of refraction n_1 and n_2. There is a planar interface between the two as shown. A ray of light propagates from point $A = (0, y_1)$ in the first medium to point $B = (L, -y_2)$ in the second, striking a point $(x, 0)$ on the interface. Obtain the time of propagation from A to B, keeping in mind that speed of propagation of light is c/n in a medium of refractive index n. Minimize your expression for the total time as a function of x and show that this leads to Snell's law. (This is Fermat's principle of least time which applies to light propagation. This was part of the inspiration for formulating the variational principle for mechanics.)



Problem 2

A particle can slide without friction on the inner surface of a hemispherical bowl (of negligible thickness) which is resting on the ground as shown. The radius of the bowl is R. Obtain the Lagrangian and equations of motion of the particle. (You should keep in mind that the particle can have angular motion as well as radial motion. The particle cannot get off the surface, so that the vertical motion is related to the radial motion.)



Problem 3

A thin wire is bent in the shape of a curve given by $z = k\,r^3/3$, where $r^2 = x^2 + y^2$. It is placed on the ground as shown and spun around the vertical axis (z-axis) with a constant angular velocity ω. A bead of mass m can slide frictionlessly along the wire.

a) Obtain the Lagrangian and the equations of motion

b) Show that there are two equilibrium points where the force on the particle vanishes.