PHY 71100: ANALYTICAL DYNAMICS

Problem Set 2

Due September 25, 2024

Problem 1

Consider the motion of a point-particle of mass m in an attractive central potential $V(r) = kr^2/2$, where k is a real positive constant. The motion can be considered to be in a plane and the Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2) - V(r)$$

Show that all orbits are bounded and that there is a minimum energy $E_{\min} = (l^2k/m)^{\frac{1}{2}}$, where l is the angular momentum.

Problem 2

A particle of mass m can move in two dimensions under the influence of a central force

$$F = -\frac{\partial V}{\partial r} = -\frac{ar}{b}\exp\left(-\frac{r^2}{2b}\right)$$

where r is the radial distance and a, b are positive constants.

a) Find the potential energy $V(r)$ corresponding to this force.

b) For what range of values of the energy is the motion bounded?

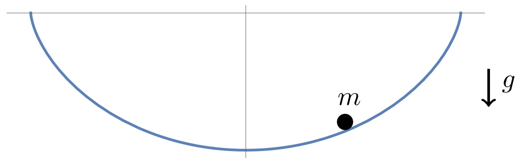
c) If an orbit is circular, what should be the relation between the angular momentum and the radius of the orbit? (You will get a transcendental equation, you do not have to solve it; just give this relation.)

Problem 3

Two beads can move frictionlessly on a vertically placed hoop of radius a. The hoop is rotating around the vertical axis at an angular velocity of ω. Use the angular displacements θ_1 and θ_2 as generalized coordinates. In addition to the gravitational potential, there is also an interaction between the beads given by the potential energy $V_{int} = \frac{1}{2}kd^2$, where d is the separation between the beads and k is a constant. Obtain the Lagrangian and the equations of motion for the system.

Problem 4

A cycloid is given parametrically by $x = a(\theta + \sin\theta),\ y = -a(1 + \cos\theta)$, where θ is an angular variable, which can be taken to be $-\pi \leq \theta \leq \pi$. A particle can oscillate around the minimum on the inner surface of a cycloid in the vertical plane, see figure. Obtain the equations of motion and the general solution. (*Hint*: Use $\sqrt{2a(y + 2a)}$ as the generalized



variable. This problem was first analyzed by Christian Huygens and played a role in early attempts to solve the *problem of the longitude*. If you do not know what the problem of the longitude is, do look it up, there is a whole fascinating history for it.)