PHY 71100: ANALYTICAL DYNAMICS

Problem Set 3

Due October 7, 2024

Problem 1

If there is an ambient cloud of dust (or dark matter) in the solar system, then a planet (of mass m) will be subject to a potential $V = \frac{1}{2}kr^2$, (k is a positive constant), in addition to the Sun's gravitational potential $-GMm/r$.

a) Write down the Lagrangian and the equations of motion for the planet in this case. (You can assume $m \ll M$.)

b) Find the radius for which a circular orbit is possible.

c) Consider a radial perturbation (to first order) of this orbit and work out the perturbation equation. Is the motion bounded in this case?

Problem 2

In class (and in my lecture notes) I worked out the scattering for a particle subject to a repulsive $1/r^2$ potential. Here we will consider an attractive central force potential

$$V(r) = -\frac{\alpha}{r^2}, \qquad \alpha > 0$$

a) Show that the particle will fall into the center for angular momenta below a critical value. Find this value.

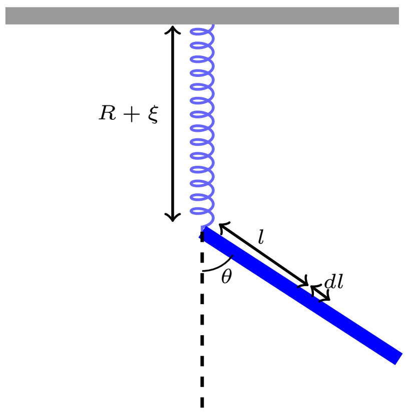
b) For energies and angular momenta for which there is no such instability, find the differential scattering cross section for particles scattering off this potential.

Problem 3

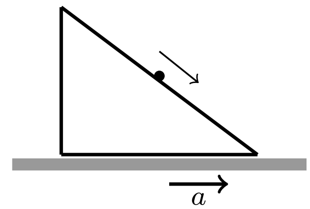
A bar of length L and uniform mass per unit length is suspended from the lower end of a vertically suspended spring, of negligible mass and spring constant k, as shown. The spring can only oscillate in the vertical direction and the motion is confined to one vertical plane, see figure. Obtain the Lagrangian and the equations of motion for the system without making any approximation of small amplitudes. (Hint: Consider a differential mass element of mass $(M/L)dl$, where M is the total mass of the bar. Obtain the kinetic and potential energies for this mass element and integrate over l from zero to L to get the energies for the bar.)

Problem 4

A particle of mass m can slide down an inclined plane under the force of gravity. This entire setup is placed inside a train (the bed of the train is the thickened line in figure) which is moving with a constant acceleration in a straight line, say, along the x-axis. Obtain the Lagrangian and the equations of motion for the particle.



Problem 3



Problem 4