PHY 71100: ANALYTICAL DYNAMICS

Problem Set 5

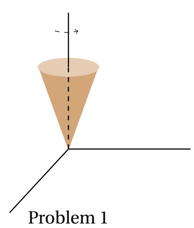
Due November 20, 2024

Problem 1

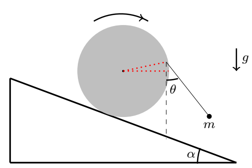
Consider a solid cone which is rotating around its axis, as shown. Calculate the moment of inertia and obtain the Lagrangian for this motion.

Problem 2

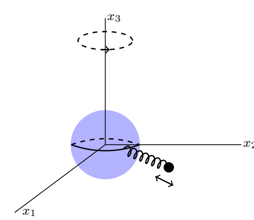
A solid circular disk of mass M and radius R can roll down an inclined plane under the force of gravity. There is an axle at the rim of the disk from which is suspended a pendulum, with a bob of mass m and a string of negligible mass. (The axle is to ensure that the string for the pendulum does not wind around the disk, but keeps a fixed length, as it rolls down.) Obtain the Lagrangian for the motion of the pendulum and the disk.



Problem 1



Problem 2



Problem 3

Problem 3

A solid spherical ball of mass M and radius R is at the center of the coordinate system as shown. At one point on the equator is attached a spring (of negligible mass) which is horizontal, in the (x, y)-plane. At the other end of the spring is a small mass m. The system can undergo rotations around the z-axis and the mass m can move by stretching (or compressing) the spring. (Ignore any other kind of motion for the spring.)

a) Obtain the Lagrangian for the system and the equations of motion. Show that there is a conserved angular momentum for the system.

b) If l_0 is the length of the unstretched spring when it is at rest, determine its equilibrium length when the system is rotating with angular momentum J, in the approximation $M \gg m$. (Moment of inertia for a sphere $= \frac{2}{5}MR^2$.)

Problem 4

Consider a thin homogeneous plate with the principal moments of inertia I_1, I_2, I_3, $I_2 > I_1 = I_2 \cos 2\alpha$ and $I_3 = I_1 + I_2$. Take the origin of the body-fixed and space-fixed coordinates as the center of mass of the plate, with (x_1, x_2)-axes along the plate and the x_3-axis perpendicular to it. At time $t = 0$, the plate starts rotating freely with angular velocity Ω about an axis at an angle α from the plane of the plate and perpendicular to the x_2-axis, so that initially $\Omega_1(0) = \Omega \cos \alpha,\ \Omega_2(0) = 0,\ \Omega_3(0) = \Omega \sin \alpha$. Show that the angular velocity about the x_2-axis is given by

$$\Omega_2(t) = \Omega \cos \alpha \ \tanh(\Omega t \sin \alpha)$$