PHY 71100: Analytical Dynamics

Problem Set 6

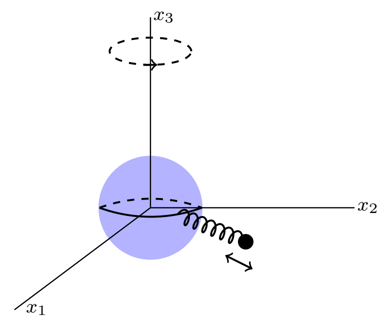
Due December 2, 2024

Problem 1

A solid spherical ball of mass M and radius R is at the center of the coordinate system as shown. At one point on the equator is attached a spring (of negligible mass) which is horizontal, in the (x, y)-plane. At the other end of the spring is a small mass m. The system can undergo rotations around the z-axis and the mass m can move by stretching (or compressing) the spring. (Ignore any other kind of motion for the spring.) The Lagrangian for this was obtained in one of the previous problem sets as

$$L = \frac{1}{2}\left(I + m(R+l)^2\right)\dot{\varphi}^2 + \frac{1}{2} m\,\dot{l}^2 - \frac{k}{2}(l - l_0)^2$$

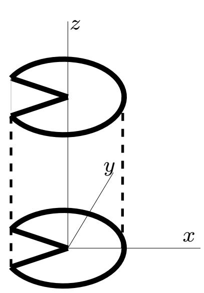
Work out the canonical momenta and the Hamiltonian. Also, obtain the canonical equations of motion.

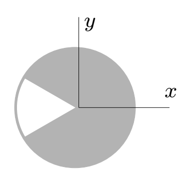


Problem 1

Problem 2

We consider a cylinder of height h, radius of cross section R, with a wedge of angle 2α cut out as shown. Obtain the moment of inertia of the object for rotations around the z-axis. (The cylinder is solid, I show the hollow picture to illustrate the coordinate system.)





Cross section of cylinder
with wedge cut out

Problem 2

Problem 3

The Lagrangian describing the motion of a particle of mass m in a frame which is rotating with angular velocity $\vec{\omega}$ was given in class as

$$L = \frac{1}{2} m \left(\dot{x}^2 - 2\, \epsilon_{ijk} \dot{x}^i x^j \omega^k + \omega^2 x^2 - (\vec{\omega} \cdot \vec{x})^2 \right) - V(x)$$

Write down the canonical momenta, the Hamiltonian and the Hamiltonian equations of motion.

Problem 4

For two particles of masses $m_1,\ m_2$ moving in one dimension, the transformation to the center of mass and relative coordinates is given by

$$P_1 = p_1 + p_2, \qquad Q_1 = \frac{m_1 q_1 + m_2 q_2}{m_1 + m_2}$$
$$P_2 = \frac{m_2 p_1 - m_1 p_2}{m_1 + m_2}, \qquad Q_2 = q_1 - q_2$$

Show that this transformation is canonical. (*Hint: Write* $(P_1, Q_1, P_2, Q_2) = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ *and* $(p_1, q_1, p_2, q_2) = (\xi_1, \xi_2, \xi_3, \xi_4)$*. Then show that the Poisson bracket of two functions, say,* f *and* g*, is the same using the* λ*'s or the* ξ*'s.*