PHY V2500: QUANTUM MECHANICS I

Problem Set 2

Due September 25, 2025

Problem 1

Consider a wave function in one dimension of the form

$$\psi = A\, e^{-i\omega t}\, x^2\, e^{-ax}$$

where A, a are constants and x is in the range $[0, \infty]$.

a) From the normalization condition $\int dx \psi^* \psi = 1$, determine the constant A in terms of a.

b) Calculate the expectation values $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$, $\langle p^2 \rangle$.

Problem 2

Consider the problem of the particle in a one-dimensional box of length L which was worked out in class. Consider a wave function

$$\psi = A\, \sin^2(n\pi x/L)$$

a) Find A from the normalization condition.

b) Expand the given ψ as a linear combination of the eigenstates of the Hamiltonian.

Problem 3

a) Determine the eigenvalues and eigenvectors of the matrix

$$M = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & e^{i\varphi} \end{bmatrix}$$

where θ, φ are some angular variables.

b) Is this matrix hermitian, unitary or neither?

Problem 4

a) Starting with the Heisenberg algebra, calculate the commutators $[\hat{x}, \hat{p}^n]$, $[\hat{p}, \hat{x}^n]$.

b) Recall the familiar identity

$$e^a\, e^b = e^{a+b}$$

Let $\hat{A}$ and $\hat{B}$ be two operators which do not necessarily commute. *Then a similar identity as given above does not hold.* A more general identity is of the form

$$e^{\hat{A}}\, e^{\hat{B}} = e^{\hat{A}+\hat{B}+F_2+F_3+\cdots}$$

where F_2 is quadratic in $\hat{A}$, $\hat{B}$, F_3 is cubic and so on. Expanding both sides to quadratic order and comparing, calculate F_2.