PHY V2500: Quantum Mechanics I

Problem Set 4

Due October 23, 2025

Problem 1

The wave function for a particular state for a one dimensional system is given by

$$\psi(x) = A\, e^{-i\omega t}\, x\, e^{-ax}$$

where x is the in the range $0 \leq x \leq \infty$. a) Determine the normalization factor A.
b) Calculate the expectation values $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$ and $\langle p^2 \rangle$.
c) Verify the uncertainty relation for x and p using your results.

Problem 2

Consider the harmonic oscillator problem discussed in class. The oscillator is initially (at time $t = 0$) given to be in the state decsribed by the wave function

$$\psi(x) = \frac{1}{\sqrt{2}}\,(u_1 + u_2)$$

where $u_0 = \langle x|1\rangle$ and $u_1 = \langle x|2\rangle$ are the wave functions for the first adn second excited states, respectively. Obtain ψ at time $t > 0$ and calculate the probability to find the particle in the range $-\infty < x < 0$. (*Caution*: You cannot use orthogonality since the required range is only the half-line.)

Problem 3

Consider the harmonic oscillator with the Hamiltonian

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2\, \hat{x}^2$$

A particle is created in the state given by the wave function

$$\psi = \begin{cases} C(1 - |x|/a) & |x| \leq a \\ 0 & |x| > a \end{cases}$$

This wave function is thus a bump around the origin vanishing beyond a on both sides. Calculate the expectation value of the Hamiltonian for this wave function.Your answer will be a function of a; let us call this as $\mathcal{E}(a)$. Minimize $\mathcal{E}(a)$ with respect to a and then use it back in $\mathcal{E}(a)$ to obtain the minimum value for $\mathcal{E}$ for this type of wave function.

Problem 4

A wave function of the form $\psi \sim e^{ikx}$ is not localized as $\psi^*\psi$ is the same for all x. The probability to find the particle is the same everywhere. An approximately localized state can be obtained as a combination of the plane wave states. A simple example of that, in one dimension, is

$$\psi = \frac{1}{(\pi\sigma^2)^{1/4}} \exp\left[iq(x-x_0) - i\frac{\hbar q^2 t}{m}\right] \exp\left[-\frac{(x-x_0-vt)^2}{2\sigma^2}\right]$$

where $v = \hbar q/m$. Here q and x_0 are free (and constant) parameters, and σ is another parameter giving the extent of localization. For this expression, calculate $\psi^*\psi$ and the probability current J. Calculate the time-derivative of $\psi^*\psi$ and express it in terms of J.