PHY V2500: QUANTUM MECHANICS I

Problem Set 6

Due November 25, 2025

Problem 1

The Hamiltonian for a rotating body is given by

$$H = \frac{1}{2\,I} \sum_j L_j L_j + \mu B\, L_3$$

where μ, B are constants; I is the moment of inertia, taken to be another constant.

a) Obtain the eigenvalues and eigenstates of this Hamiltonian using angular momentum theory.

b) What are the degeneracies for the energy eigenstates?

c) Obtain the time-evolution of $\langle L_i \rangle$ (for all i) using

$$i\hbar \frac{\partial \langle L_i \rangle}{\partial t} = \langle L_i H - H L_i \rangle$$

Problem 2

In the $|nlm\rangle$ notation, the wave functions for the electron in the Hydrogen atom for $n = 2$ are given by

$$\begin{aligned} \langle \vec{x}|210\rangle = \psi_{210} &= \frac{1}{\sqrt{32\pi}} \frac{r}{a^{\frac{5}{2}}}\, e^{-r/2a}\, \cos\theta \\ \langle \vec{x}|21\pm 1\rangle = \psi_{21\pm1} &= \mp \frac{1}{\sqrt{64\pi}} \frac{r}{a^{\frac{5}{2}}}\, e^{-r/2a}\, \sin\theta\, e^{\pm i\varphi} \end{aligned}$$

where $a = \hbar^2/me^2$ is the Bohr radius. For each of these wave functions calculate the expectation value of $\sin\theta$, $\sin^2\theta$ and r.

Problem 3

A nucleus which is rotating is described by the Hamiltonian

$$H = \frac{L_1^2 + L_2^2}{2I_1} + \frac{L_3^2}{2I_3}$$

where $\vec{L}$ is the orbital angular momentum and I_1, I_2 are two of the principal moments of inertia, with $I_1 \neq I_2$. (They are not operators.) Obtain the eigenstates and eigenvalues of this Hamiltonian.

Problem 4

The time-derivative of the expectation value of the angular momentum is given by

$$i\hbar \frac{\partial}{\partial t} \langle L_i \rangle = \Big\langle L_i\, H - H\, L_i \Big\rangle$$

where the angular brackets indicate the expectation value for some state. Consider a particle with electric dipole moment $\vec{\mu}_1$ interacting with a fixed dipole of dipole moment $\vec{\mu}_2$. The Hamiltonian for dipole 1 is given by

$$H = \frac{p^2}{2M} - \frac{3\,\vec{x}\cdot\vec{\mu}_1\,\vec{x}\cdot\vec{\mu}_2}{r^5} + \frac{\vec{\mu}_1\cdot\vec{\mu}_2}{r^3}$$

Calculate the required commutator to obtain the rate of change of angular momentum. This is the quantum version of the usual torque equation in mechanics.